Unit 2

2.1 Slope and rates of change

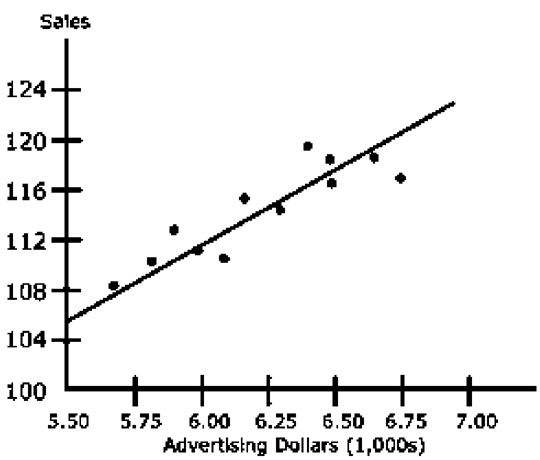
2.2 Equations of lines

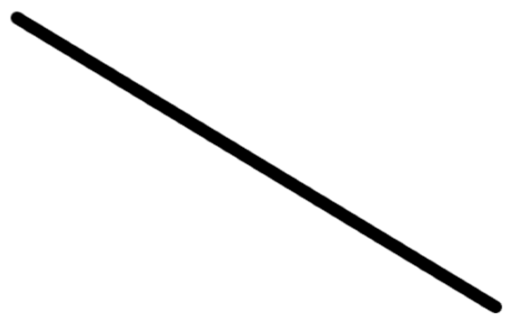
2.3 Linear Regression

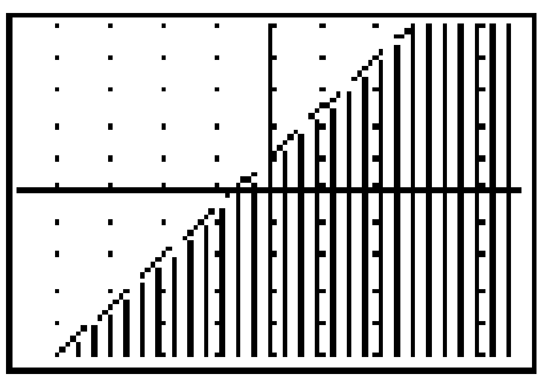
2.4 Solving linear equations and inequalities graphically

2.5 Linear inequalities and systems of linear inequalities

2.6 Systems of linear equations





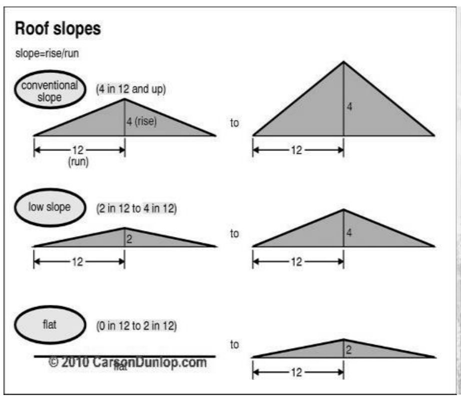


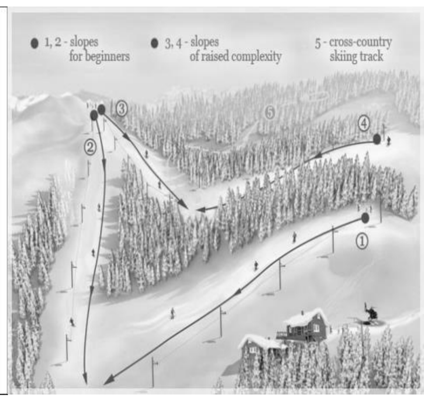
2.1 Slope and Rates of Change

Slope is a measure of change. It describes how one variable changes in relation to the change in another variable. Slope can also be a measure of inclination or slant. Slope can be found verbally, numerically, graphically and algebraically. The letter “m” is used to represent slope.

Verbally:

The word slope is used when discussing ski slopes. In this instance, it is used because of the inclination of the hills for skiing.   The word grade is used when the inclination of roads is being described. The slope of a roof is called the pitch.





Otherwise, the units of measure will help identify slopes in context. Slope describes how two variables change in relation to each other. The word “per” is usually used when describing slopes in context. Also, slope is usually a change in output in relation to a change in the input. $m = \frac{change\ in\ output}{change\ in\ input}$

Examples:

1. Hourly wages: $8.50 per hour means that for every increase of one hour a person will earn an additional $8.50.

2. Gas mileage: 23 MPG or 23 miles per gallon means that for each gallon of gas a vehicle will travel an additional 23 miles.

3. Food costs: $1.39 per pound means that for each additional pound purchased the cost will increase by $1.39.

4. Grade of a road: 18% = $\frac{18}{100}$ means that for every 100 horizontal feet of the road the vertical change in the road is 18 feet.

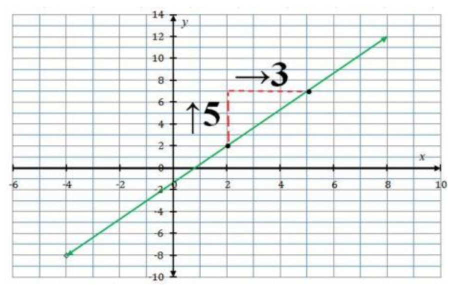
Graphically: Slope is the vertical change in relation to the horizontal change. Recall that the vertical axis represents the output variable and the horizontal axis represents the input variable. Then, slope can be described as $m = \frac{vertical\ change}{horizontal\ change}$ or $m = \frac{rise}{run}$.

To calculate slope on a graph, identify any two points on the graph and then find the vertical change or rise between the two points divided by the horizontal change or run between those two points. Movement in the upward or right directions are positive changes; whereas, movement downward or to the left are negative changes.

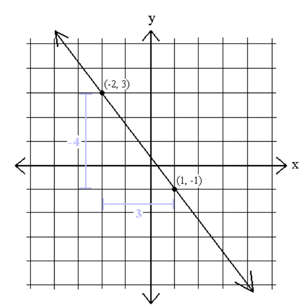
Lines with a positive slope will slant upward from left to right and lines with a negative slope will slant downward. Lines with zero slope are horizontal lines. Vertical lines have an undefined slope or no slope.

Examples:

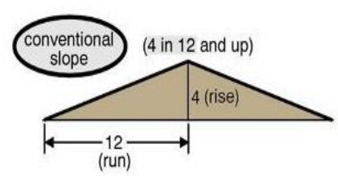
1. A slope of $\frac{5}{3}$ is shown on the graph to the right. Choosing the two points marked on the graph, the vertical change is 5 and the horizontal change is 3. Try choosing two other points, you will find that the slope is the same.



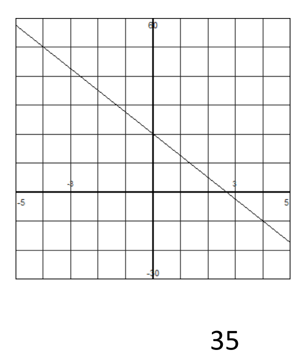
2. The slope of this line is m = $\frac{-4}{3}$. Note that the line slants downward from left to right. The diagram calculates the slope from the point in the upper left to the point in the lower right. If you calculated the slope starting with the point in the lower right, you would find that the slope is m = $\frac{4}{-3}$. Both of these slopes are equal and both are the same as m = $-\frac{4}{3}$.



3. Looking at the picture of the roof. The conventional slope or pitch of a roof is m = $\frac{4}{12} = \frac{1}{3}$.



4. The slope of the graph shown is m = $-\frac{30}{4}$. The scale on the vertical axis is 10 units and the scale on the horizontal axis is 1 unit. Using the two points (0, 20) and (4, -10), the vertical change is -30 units and the horizontal change is 4 units. Be sure to look at the scales on the axis and not just count the units.



Numerically: Finding the slope from a table of values is finding the slope numerically. To find the slope from a table, choose two points in the table and then find the change in the output variable and the change in the input variable.

Examples:

1. Given the table shown, find the slope including appropriate units.

T, hours	D, miles
0	100
1	125
2	150
4	200

Using the first two points listed in the table, we can find the change in distance as 25 miles and the change in time as 1 hour. Therefore, the slope is m = $\frac{25\ miles}{1\ hour}$ or 25 MPH.

	T, hours	D, miles	
1	0	100	25
	1	125	
	2	150	
	4	200	

If you chose to use the last two points in the table, the change in distance is 50 miles and the change in time is 2 hours, so m = $\frac{50\ miles}{2\ hour}$ or 25 MPH.

2. Sam received a $25 gift card for Redbox movies for his birthday. The value of the gift card is shown in the table below. Find the slope and interpret it in context.

N, number of movies rented	V, value of card in dollars
0	25
5	19
15	7

Choosing the first two points in the table, the value of the card decreases by $6 and the change in the number of movies is 5. So the slope is m = $\frac{-6}{5} = -1.20$. Therefore, for each movie rented the value of the card decreases by $1.20 or it costs Sam $1.20 per movie.

Symbolically: The formula for slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$. To find slope using the formula, identify two points and find the difference in the y-values divided by the difference in the x-values.

Examples:

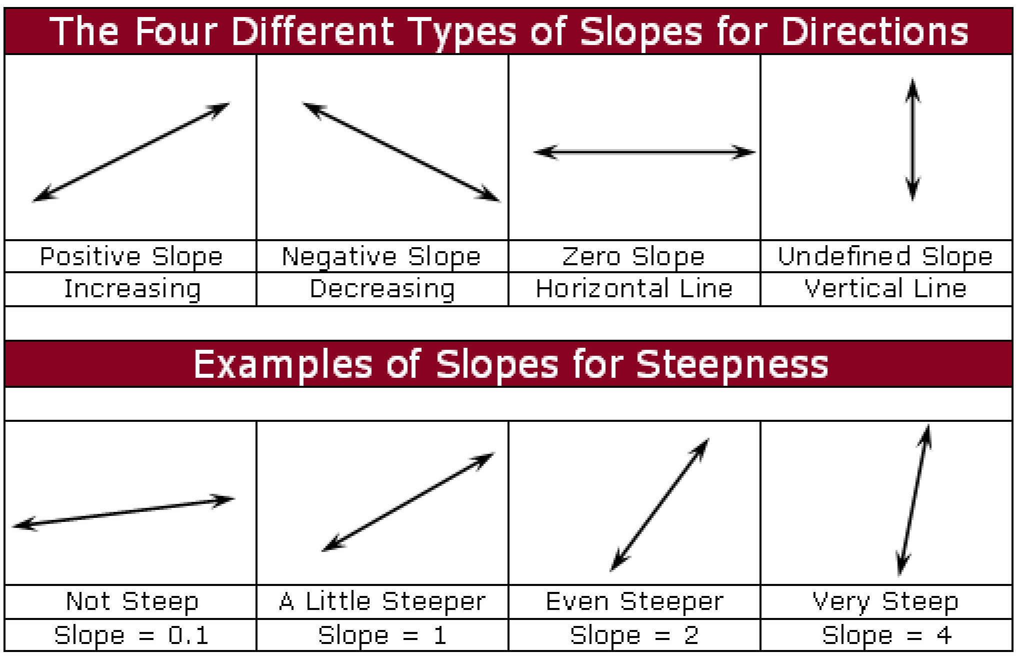
1. Find the slope between the points (8, -1) and (6, 9).

$m = \frac{9-(-1)}{6-8} = \frac{10}{-2} = -5$

2. Three pounds of apples costs $8.37 and 7 pounds of apples costs 19.53. Find the slope and interpret the slope in context.

   Assuming that the cost (output) relies on the number of pounds purchased (input), then we are given the two points (3, 8.37) and (7, 19.53) and the slope can be found by

$m = \frac{19.53-8.37}{7-3} = 2.79$. In context, the apples cost $2.79 per pound.



Homework:

Find the slope within the following scenarios including units.

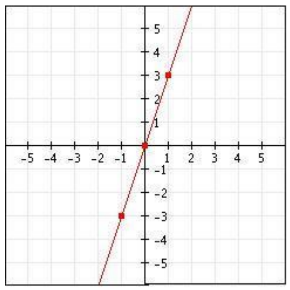
1. Paul walks 10 miles every morning at an average speed of 3.5 miles per hour.

2. Henry's car holds 12 gallons of gasoline. He fills up the tank at a cost of $3.59 per gallon.

3. A gym membership has an initial fee of $59 plus additional costs of $15 per month.

Find the slope of the graphs shown, include units if appropriate.

4.



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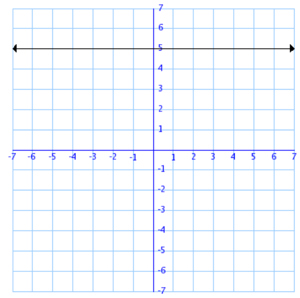

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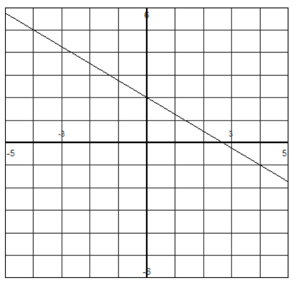

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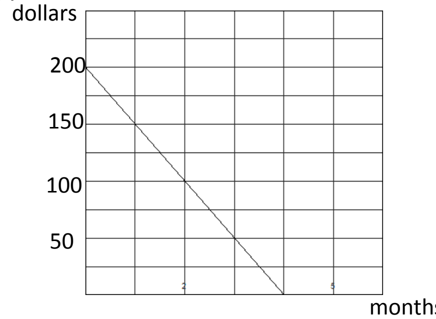


Find the slope of the information in the tables, include units if appropriate.

8.

x	-1	0	1	2
y	-5	3	11	19

9.

x, year	1970	1980	1990	2000
y, population	4000	5500	7000	8500

10.

t, minutes	0	1	2	3
h, height in feet	120	90	60	30

11.

x, hour	0	2	4	6
y, dollars	0	24	48	72

Find the slope of the line passing through the given points.

12. (1, 5) , (7, 18)

13. (-3, -7), (0, -2)

Find the slope of the line passing through the given points.

14. (9, 0), (9, 2)

15, (-6, 1), (4, 1)

16. (3.2, 1.5), (4.1, -0.2)

Modeling: 17-20 Choose the graph (a-d) that models the situation best.

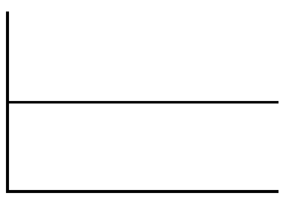
17. The cost of a party that has a rental fee of $40 and a fee of $10 per person.

18. The distance between Orlando, FL and Atlanta, GA.

19. The distance a train travels when it is going 60mph.

20. The amount on a $25 prepaid phone card as phone calls are made at a rate of $0.04 per minute.

a.



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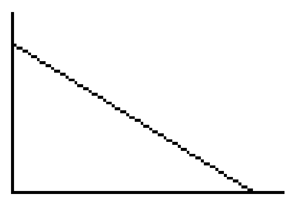

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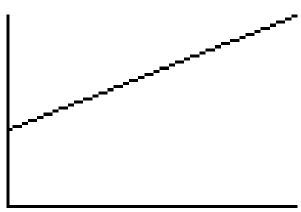

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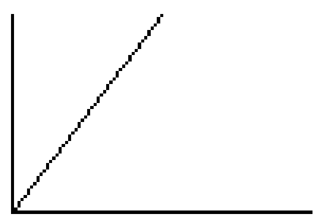


2.2 Equations of lines

A linear equation is a first-degree equation. There are 3 forms of the equation of a line.

Slope-intercept form: Slope-intercept form is y = mx + b where m is the slope of the line and the point (0, b) is the y-intercept of the line.

For example:

1. y = ½ x + 5 is a line with a slope of ½ and a y-intercept of (0,5).
2. y = -4x is a line with a slope of -4 and a y-intercept of (0,0).

Point-slope form: Point-slope form is $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is any point on the line.

For example:

1. y – 4 = 3(x -1) is a line with a slope of 3 and a point at (1, 4).
2. y – 6 = $-\frac{2}{3}$(x + 4) is a line with slope $-\frac{2}{3}$ and the point (-4, 6).

Standard form: Standard form looks like ax + by = c where all terms with variables are on one side of the equation and the constant is on the other. The slope or points would need to be calculated from this form. This form will sometimes be used in application problems.

For example: Taylor goes to Taco Bell and buys tacos for $1.29 each and burritos at $1.79 each. She spends $10.53. Write an equation for the amount spent in terms of the number of tacos and burritos she bought.

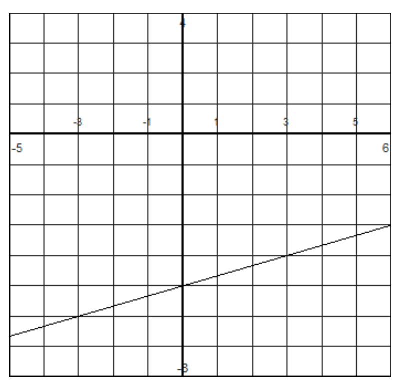
An appropriate equation would be 1.29t + 1.79b = 10.53 where t is the number of tacos and b is the number of burritos she bought.

Both of the first two forms can be used to write the equation of a line given a graph, points, or a table of values. The following examples will find the equation of the lines using first slope-intercept form and then point-slope form.

Examples:

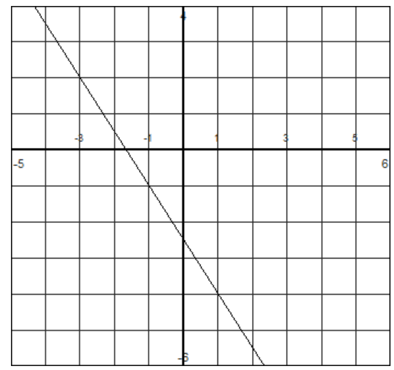
1. Find the equation of the line shown in the graph.

   A. Using slope-intercept form: We can see from the graph that the y-intercept is is (0, -5) and the slope is $\frac{1}{3}$. So, the equation of the line is y = $\frac{1}{3}x - 5$.



B. Using point-slope form: The slope is $\frac{1}{3}$. Choose any other point on the graph such as (3, -4) and plug into the equation. $y - (-4) = \frac{1}{3}(x - 3)$ or $y + 4 = \frac{1}{3}(x - 3)$. You may choose any point that is on the graph.

2. Find the equation of the line shown in the graph.



A. Slope-intercept form: The slope of the line is $-\frac{3}{2}$ but the y-intercept is not obvious from the graph. In this case, choose any point on the graph and use the equation to find the value of b.

Choosing the point (-3, 2):

y = mx + b Plugging in the value for m and the coordinates of the chosen point gives:

$2 = -\frac{3}{2}(-3) + b$

$2 = \frac{9}{2} + b$

$b = -\frac{5}{2}$

So, the equation of the line is $y = -\frac{3}{2}x - \frac{5}{2}$.

B. Point-slope form: The slope is $-\frac{3}{2}$ and a point on the graph is (-3, 2) so an equation for the line would be $y - 2 = -\frac{3}{2}(x + 3)$.

3. The table below shows the height of an object after t minutes have elapsed. Write the equation of the line.

t, minutes	0	1	2	3
h, height in feet	120	90	60	30

A. Slope-intercept form: The slope is -30 feet per minute. The y-intercept is (0, 120). The equation of the line is h = -30t + 120.

B. Point-slope form: The slope is -30 feet per minute and a point is (0, 120) so an equation would be y – 120 = -30 (t – 0).

4. Three pounds of apples costs $8.37 and 7 pounds of apples costs 19.53. Find the linear equation for the price of apples in terms of the number of pounds of apples purchased.

The slope is $2.79 per pound. Using the point (3, 8.37), an equation would be

C – 8.37 = 2.79(p – 3) which simplifies to C = 2.79p.

Equations of horizontal and vertical lines:

A horizontal line has a slope of zero. The equation has the form y = b.

For example, the equation of a horizontal line through the point (0, 4) is y = 4.

A vertical line has an undefined slope. The equation has the form x = a.

For example, the equation of a vertical line through the point (5, -3) is x = 5.

Parallel and Perpendicular Lines

Parallel lines are lines that never intersect. Parallel lines have the same slope.

Perpendicular lines are lines that intersect at right angles or 90° angles. Perpendicular lines have opposite reciprocal slopes or slopes that multiply to -1. For example, lines with a slope of 2 and – ½ will be perpendicular.

Examples:

1. Find a line that is parallel to y = ¾ x + 1 through the point (6, 0).

   Parallel lines have the same slope so the slope of the line must also be ¾ . Using point-slope form the equation will be y – 0 = ¾ (x – 6) or y = ¾ x – 9/2.

2. Find a line that is perpendicular to y = 2/3 x – 5 through the point (1, -4).

   Perpendicular lines have opposite reciprocal slopes. The slope of the line given is 2/3 so the slope of the perpendicular line must be -3/2. Using the point slope equation, the equation of the new line will be y + 4 = -3/2(x – 1).

Homework:

Write the equation of the line.

1. With slope of -7 and the point (0, -1)
2. With slope of m = $\frac{2}{7}$ and the point (0, 2)
3. With slope of m =-$\frac{5}{2}$ and the point (3, 1)
4. Through the points (4, 7) and (-1, 5)

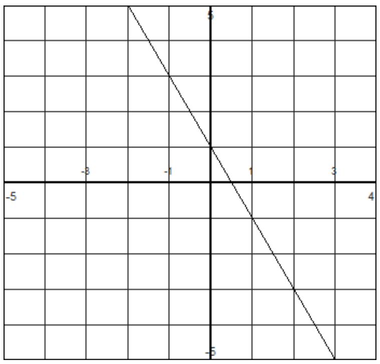
5. Through the points (6, -2) and (0, 3)

6. With a slope of 0 and the point (5, 4)

7. Vertical line through the point (3, -6)

8. Horizontal line through the point (1, 5)

9.



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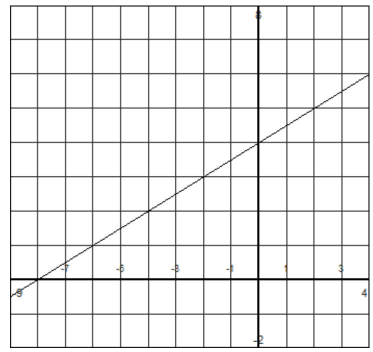

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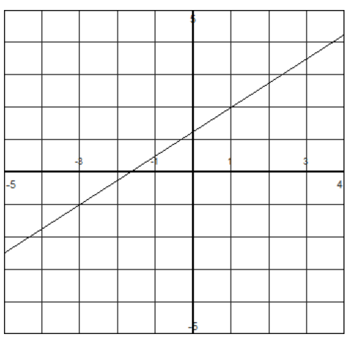

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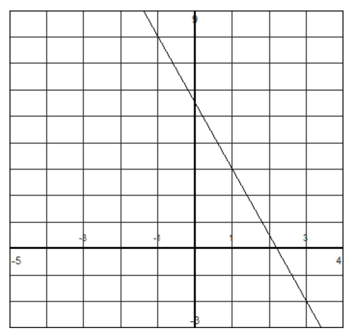

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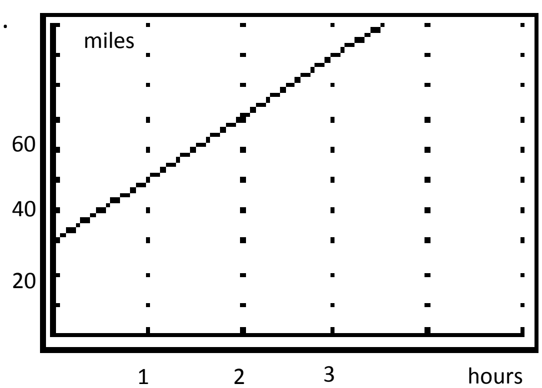

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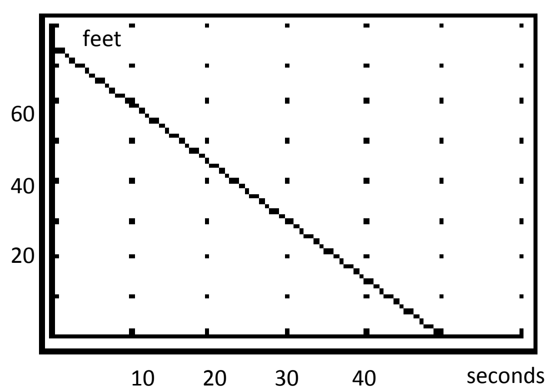


15.

t, minutes	0	1	2	3
h, height in feet	120	90	60	30

16.

x, hour	0	2	4	6
y, dollars	0	50	100	150

17.

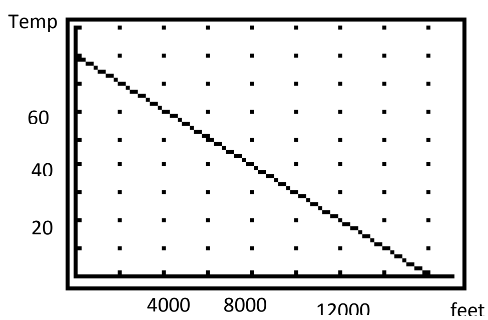
x	-3	0	3	6
y	10	12	14	16

18. In 2002, twenty-one percent of married couples in the US lived in single-earner households compared with 63 percent in 1950. Write an equation of a line which finds the percent of single-earner households in terms of the year.

19. A 2-lb box of candy costs $21 and a 5-lb box of candy costs $46.50.
    A. Find the slope and interpret.
    B. Find the vertical intercept and interpret.
    C. Write an equation of the line which gives the price of the box of candy in terms of its weight.

20. The median age of the US population from 1820-1995 was modeled by f(x) = 0.09x-147.1, where x represents the year.
    A. What is the slope of the line and what does it mean?
    B. What is the vertical intercept and what does it mean?

21. The figure below shows the air temperature at certain altitudes. An altitude of 0 feet is sea level. The altitude scale is 2000 feet and the temperature scale is 10 degrees.

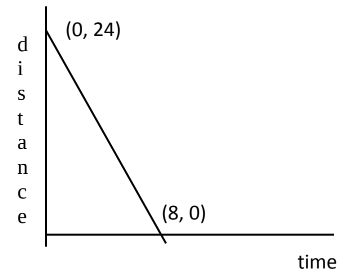


A. Calculate the slope of the graph. Explain what the slope means in the context of this problem.

B. Find the vertical intercept. Explain what this point represents in the context of this problem.

C. Write a linear equation that describes the temperature T, in terms of the altitude, A.

22. Consider the graph below. Time in seconds is graphed on the x-axis and distance in feet is graphed on the y-axis. The graph shows Alisa's distance from a pinball machine as a function of time. Which sentence is a good match for the graph?



A. Alisa stood 8 feet from the pinball machine and moved toward it, reaching it after 24 seconds.
B. Alisa stood 24 feet from the pinball machine and moved toward it at a rate of 4 feet per second.
C. Alisa stood 24 feet from the pinball machine and moved toward it, reaching it after 8 seconds.
D. Alisa stood 8 feet from the pinball machine and moved away from it, stopping when she was 24 feet away.

23. Which of the tables shown describes the graph in problem 22?

A)

time	distance
0	24
2	16
4	8
6	0
8	0

B)

time	distance
0	24
2	18
4	12
6	6
8	0

C)

time	distance
0	24
2	18
4	11
6	3
8	0

24. Consider the following table and choose the sentence that best describes the table.

A. Kirk walked away from the water ride at a rate of 6 feet per second.
B. Kirk was 42 feet away from the water ride and walked toward it at a rate of 7 feet per second.
C. Kirk was 42 feet away from the water ride and walked toward it at a rate of 6 feet per second.
D. Kirk walked away from the water ride at a rate of 7 feet per second.

Time	Distance from the ride
0	42
1	36
2	30
3	24
4	18
5	12
6	6
7	0

25. Match each of the following equations to the appropriate sentence.

A. Kirk walked away from the water ride at a rate of 6 feet per second.
B. Kirk was 42 feet away from the water ride and walked toward it at a rate of 7 feet per second.
C. Kirk was 42 feet away from the water ride and walked toward it at a rate of 6 feet per second.
D. Kirk walked away from the water ride at a rate of 7 feet per second.

I) $y = 42 - 6x$ II) $y = 42 - 7x$ III) $y = 6x$ IV) $y = 7x$

26. A newspaper publishing company has fixed costs of \$100 for storage and delivery. They have additional costs of \$0.50 per paper published.

A. Fill in the table below for costs of publishing the given number of papers.

Number of papers	Total cost
200	
500	
1000	

B. Write a linear equation that relates the total cost to the number of papers published.

C. Sketch a graph for this problem situation.

D. Give an appropriate viewing window for this problem.

27. Find the equation of the line parallel to $y = 5x + 3$ through the point (0, 1).

28. Find the equation of the line parallel to $y = -1/2\ x + 4$ through the point (-8, 2).

29. Find the equation of the line parallel to $y = 6$ through the point (4, -3).

30. Find the equation of the line perpendicular to $y = \frac{1}{4}x - 9$ through the point (2, 1).

31. Find the equation of the line perpendicular to $y = -x + 3$ through the point (-4, 7).

32. Find the equation of the line perpendicular to $y = 9$ through the point (-6, -4).

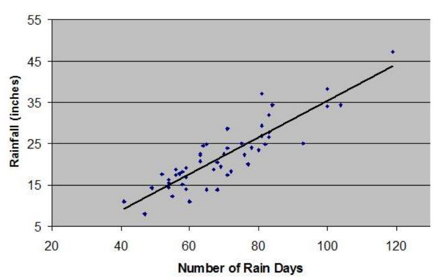
2.3 Linear Regression

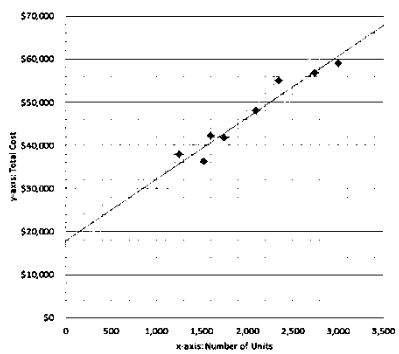
Linear regression is used to predict values from data. Linear regression uses a line of best fit to describe data that is approximately linear.

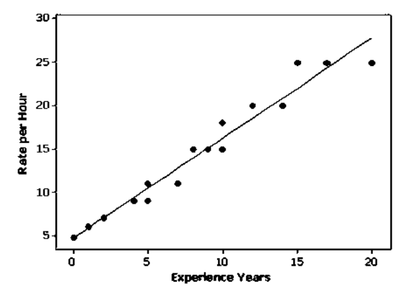
Steps for linear regression:

1. Make a graph of the data.
2. Check to see if the data looks linear. It does not need to be in a perfectly straight line but should follow a linear pattern.
3. Using a ruler, draw a line that is as close as possible to all the points on the graph. The line does not need to go through any of the points.
4. Choose two points that are on the regression line using the grid.
5. Write the equation of the line from the chosen points.

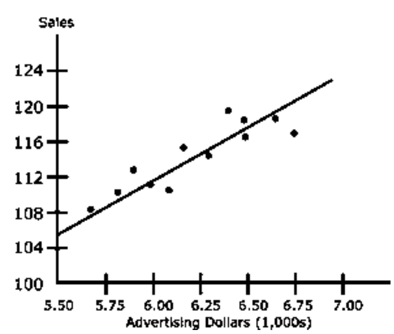
Shown below are some examples of regression lines.







Example: The graph shows the sales of a product based on the amount of advertising dollars spent. Find the equation of the regression line shown and use the equation to predict the sales if $7000 is spent on advertising.



The graph appears to have the points (5.5, 106) and (6.5, 116) on the line. To write the equation, we need to find the slope.

m = $\frac{116-106}{6.5-5.5} = 10$ sales for each $1000 spent on advertising dollars. The equation of the line is s – 106 = 10 (D – 5.5) or s = 10D + 51.

The line can be used to predict the sales for $7000 in advertising dollars by letting D = 7. s = 10(7) + 51 = 121 so the sales are predicted to be 121 products.

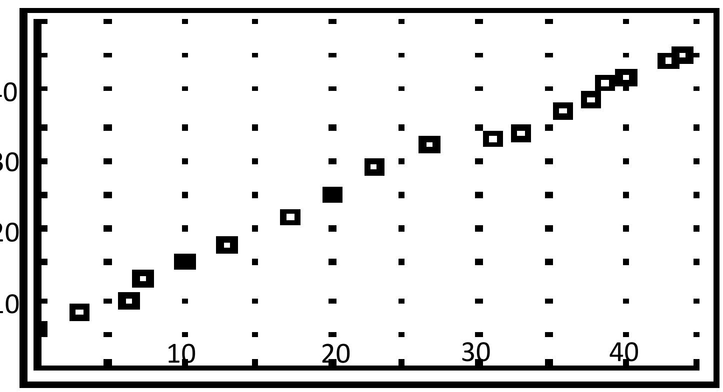
Example:

As reported in the Orlando Sentinel, stamp prices for the years 1968 until 2008 are as shown in the table below.

year	price	year	price	year	price
1968	6	1985	22	2004	37
1971	8	1988	25	2006	39
1974	10	1991	29	2007	41
1975	13	1995	32	2008	42
1978	15	1999	33	2011	44
1981	18	2001	34	2012	45

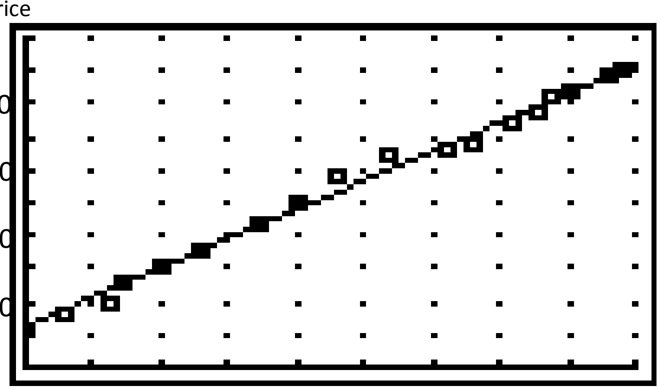
1. Plot these points. Let t = 0 represent the year 1968.

price



Time in years after 1968

price



Time in years after 1968

2. Draw a line of best fit or the regression line.

3. List two points that are on the line you drew.

   (10, 15) and (15, 20)

4. Find the slope of your line and explain what the slope represents in terms of this problem situation. m = $\frac{20-15}{15-10} = \frac{5}{5} = 1$ which means that the price of stamps rise on average 1 cent per year.

5. Find a linear equation for your line.

   P – 15 = 1(t – 10) or P = t + 5

6. Using your equation, predict the price of a stamp in 2013. 2013 is 45 years after 1968 so P = 45 + 5 = 50 or the price of a stamp is predicted to cost $0.50 in 2013.

Linear Regression on the Graphing Calculator

According to statistics, the line of best fit is the least-squares regression line which is the line which makes the vertical distances from the data points to the line as small as possible. To find this line by hand would be difficult. The graphing calculator has a built-in function that will find the least-squares regression line. A special note must be made to not just use this function until you have verified that a linear equation will be a good fit for the data.

Steps to linear regression on the calculator:

1. Press the **STAT** key. Then **1: Edit**.

2. Type the x-values of your coordinate points into one list and the y-values of your coordinate points into another list. Be sure that the x-coordinate and y-coordinate of a point are in the same row of the lists. The lists should be exactly the same length.

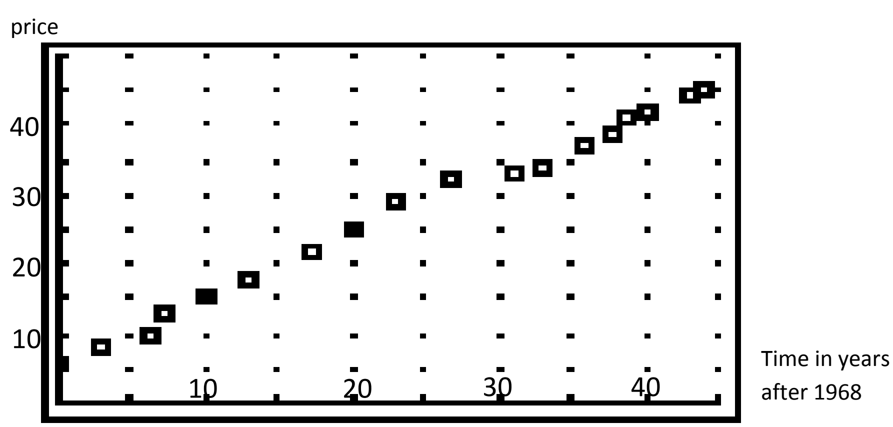
3. To look at the scatterplot:
   Press **2nd** **Y=** for STAT PLOT
   Choose a **Plot**. Turn it **ON**. The first type is a scatterplot.
   In XLIST: enter the list where you stored the x-values of your coordinate points
   In YLIST: enter the list where you stored the y-values of your coordinate points
   Set a window appropriate to the problem and press **GRAPH**.

4. If a line will provide a good fit for the data, then Press **STAT**. Choose **CALC**. Choose **4: LinReg(ax+b)**. Be sure the appropriate lists for your data are entered. In Store RegEQ: enter $\mathbf{Y_1}$. Then choose **Calculate**.

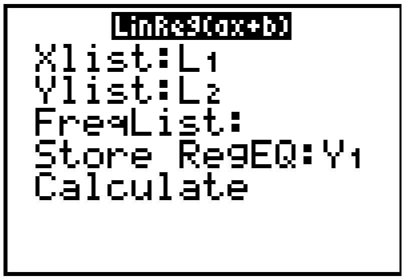
5. You will now have a screen that has **y = ax + b** with the values of **a** and **b** listed, where a is the slope of the regression line and b is the y-intercept of the regression line. You may also have on this screen a value for r^2 and r. The letter r represents the correlation coefficient which measures how good of a fit the least-squares regression line is for the data. Also, the regression equation is now stored in Y_1 on your calculator so if you go back to the graph you can see how well the line fits the data on the scatterplot.

Returning to the previous example:

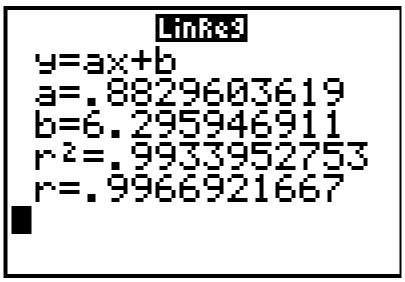
Once you enter the data into the lists, you can make a scatterplot. The scatterplot for the stamp data is shown.



Choose **STAT** and **CALC** and **5:LinReg(ax + b)**. The screen should look like the one shown below if you used lists 1 and 2 for your x and y coordinates.



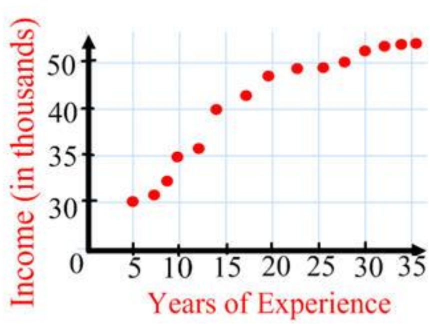
Choosing **Calculate** should show the following screen. Your screen may or may not have the values for r^2 and r.

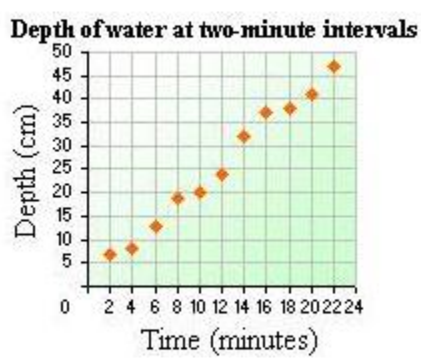


According to the calculator, the least-squares regression line for the stamp data is y = 0.88296x+6.2959. Therefore, the slope of approximately 0.88 tells us that the average rate of change of the price of stamps over those years was 0.88 cents per year. As we set t = 0 to represent the year 1968, the y-intercept of this line means that in 1968 the price of a stamp was approximately 6 cents.

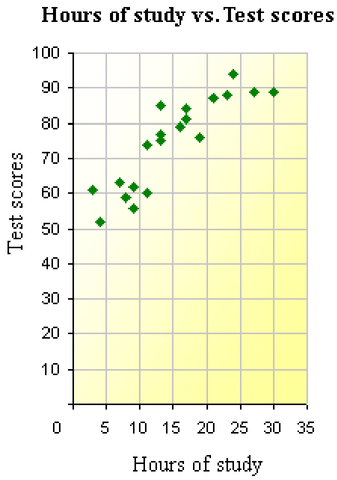
Homework:

For the following, sketch the regression line and find the equation of your line.

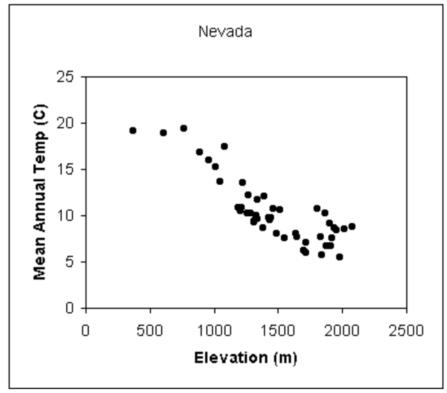
1. 

2. 

3.



4.



5. The table gives the attendance at Disney World parks in millions as reported in the Orlando Sentinel on August 23, 1998.

   A. Plot these points. **Let t = 0 represent the year 1980.**
   B. Use a straight edge to draw a line that "fits" the data.
   C. List two points that are on the line you drew.
   D. Find the slope of your line. Explain what the slope represents in terms of this problem situation.
   E. Find a linear equation for your line.
   F. Using your equation, estimate the attendance in the year 2007.
   G. Use your calculator to calculate the least-squares regression line.

year	Admissions (millions)	year	Admissions (millions)
1980	13.8	1989	28.2
1981	13.2	1990	32.8
1982	12.6	1991	28.2
1983	22.7	1992	29.6
1984	21.1	1993	29.1
1985	21.9	1994	27.6
1986	23.9	1995	32.8
1987	27.0	1996	34.4
1988	25.5	1997	36.3

For problems 6- 8:

   A. Plot the points.
   B. If the data appears linear, sketch a regression line. If not, explain.
   C. Find the equation of the regression line.
   D. Estimate the value in 2013.

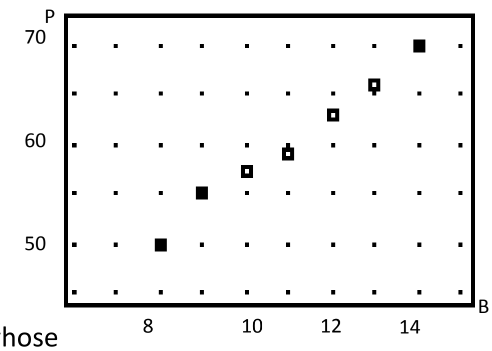
6. A study found that eighty percent of Americans were overweight in 2002, compared with 76 percent in 1998, 69 percent in 1994, 64 percent in 1990, and 59 percent in 1986.

7. The federal minimum wage in 1940 was $0.25, in 1968 it was $1.60, in 1980 it was $3.10, in 1981 it was $3.35, in 1997 it was $5.15, and in 2012 it was $7.25.

8. The number of Bachelor degrees awarded in 1970 was 792317; in 1980 the number was 929417; in 1990 the number was 1051344; in 2000 the number was 1244000; and in 2010 there were 1399542 Bachelor degrees awarded.

9. A temperature of 50°F is equal to a temperature of 283°Kelvin. Also, a temperature of 77°F is equal to a temperature of 298°Kelvin. Write a linear equation for the Kelvin temperature, K, in terms of the Fahrenheit temperature, F.

10. The scatterplot shows the approximate relationship between the height of a person, P, and the length of the humerus bone, B (the bone located between the elbow and the shoulder). The scale on the horizontal axis is 1 inch (starting at 6) and the scale on the vertical axis is 5 inches (starting at 45).



A. Use a straight-edge to draw in a line that "fits" the data.

B. Use your line to predict the height of a person whose humerus bone is 7 inches and the height of a person whose humerus bone is 11 inches.

C. Use your answers from part B to find the equation of the line that you drew.

11. The following data set gives the average heights and weights for American women aged 30-39 (source: The World Almanac and Book of Facts, 1975).

inches	58	60	62	63	64	65	66	68	70	71	72
pounds	115	120	126	129	132	135	139	146	154	159	164

A. Make a scatterplot of the data.

B. Use your calculator to calculate the least-squares regression line.

C. What is the slope of the regression line? Explain what the slope represents in terms of this problem situation.

D. What is the y-intercept of the regression line? Does this make sense in context?

E. Using the equation, estimate the weight of the average American woman who is 61 inches tall.

12. The following table shows the number of employees of Walt Disney World. (source: Orlando Sentinel, June, 24, 1998)

year	1971	1976	1980	1985	1988	1991	1995	1997	1998
employees	8000	9700	12500	16000	24000	33500	38600	48253	51500

A. Make a scatterplot of the data.

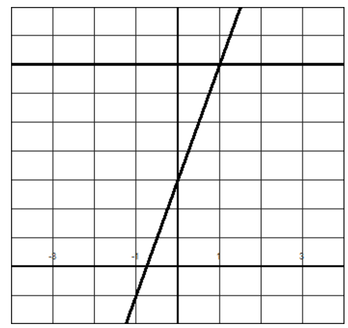
B. Is the data linear? If so, calculate the least-squares regression line. If not, just say it is not linear.

2.4 Solving Linear Equations and Inequalities Graphically

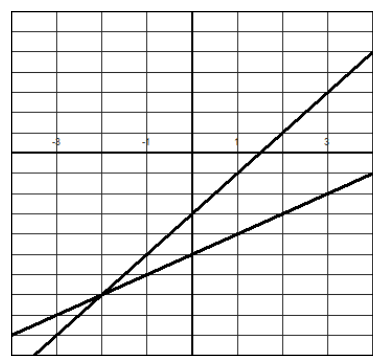
To solve a linear equation graphically, sketch a graph of the left and right sides of the equation and find the intersection of the graphs. To solve a linear inequality graphically, first find the intersection point and then determine the x-values for which the graph satisfies the inequality. If a graph is greater than another graph, the graph will lie above the other graph. If a graph is less than another, then the y-values will be below those on the other graph.

Examples:

1. Solve $4x + 3 = 7$ graphically.
   Step 1: Graph the line $y = 4x + 3$
   Step 2: Graph $y = 7$.
   Step 3: Find the x-coordinate of the intersection point.
   The solution is $x = 1$.



2. A. Solve $2x – 3 = x – 5$ graphically.
   Step 1: Graph $y = 2x – 3$
   Step 2: Graph $y = x – 5$
   Step 3: Find the x-coordinate of the intersection point.
   The solution is $x = -2$.



B. Use the graph shown to solve $2x – 3 \geq x – 5$.

The intersection point is at $x = -2$ and we need to find where the graph of $y = 2x – 3$ is above (greater than) $x – 5$. Looking at the graph this occurs where $x > -2$. The solution is $x \geq -2$.

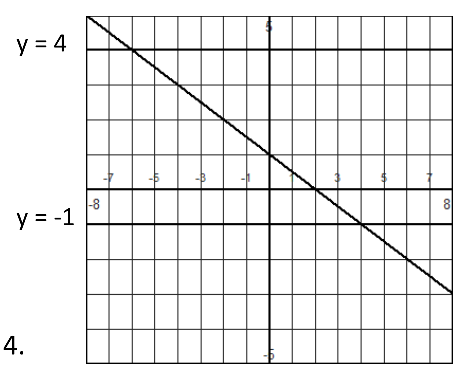
3. Given the graph of $y = f(x)$ as shown, solve:

   A. $y = 4$

   To solve, draw a line at $y = 4$, the intersection of this line and the line shown is at $x = -6$.

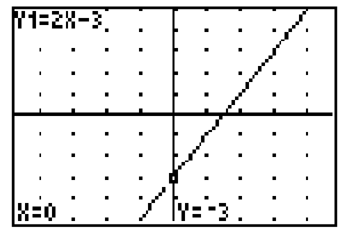
   B. $f(x) < -1$

   To solve, draw a line at $y = -1$, the intersection of this line with the graph is at $x = 4$. The graph is below the line $y = -1$ for x-values greater than $x = 4$. The solution is $x > 4$.



The graphing calculator can be used to solve equations graphically by using the intersection command which is under the CALC menu.

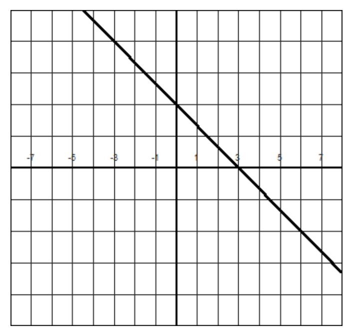
Homework:



1. A. Use the graph of y = 2x – 3 to solve the equation 2x – 3 = 3.

   B. Use the graph to solve 2x – 3 ≥ 3.

2. Use the graph of y = -2/3 x + 2 to solve:

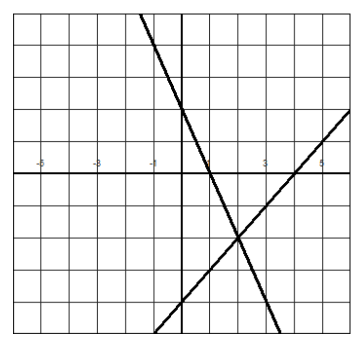


   A. -2/3 x + 2 = 4

   B. -2/3 x + 2 = 0

   C. -2/3 x + 2 < -2

3. A. Use the graph to approximate the solution to -2x + 2 = x - 4. Indicate on the graph how you found your solution.



   B. Use the graph to solve -2x + 2 > x – 4.

4. Use the graphing calculator to find the solution to $\frac{2}{3}x - 1 = -\frac{3}{4}x + 3$ graphically.

5. Use the graphing calculator to find the solution to -2x – 1 = -1/3 x + 4 graphically.

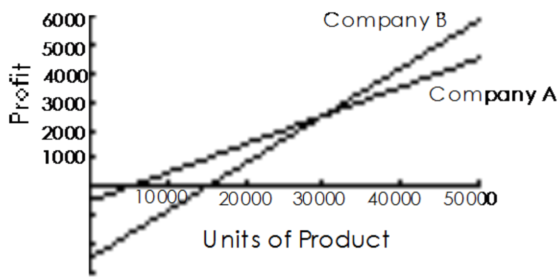
6. Sketch a graph and solve x -2 = 5/2 x + 4.

7. Sketch a graph and solve 3x + 5 = 2x – 1.

8. Sketch a graph and solve 5x + 1 ≤ 3x + 5.

9. Use a graphing calculator to find the solution to 3x – 11 > ½ x + 3.

10. The graph below represents the profit for Companies A and B if they sell x units of their product.



A. Complete the following table by using the graph to estimate.

Amount of Product	Company A Profit	Company B Profit
20,000		
45,000		

B. Which company has the larger profit when 20,000 units are produced?

C. Which company has the larger profit when 45,000 units are produced?

D. Approximate from the graph, how many units must be sold so that the two companies' profits are equal. What is the profit?

E. How many units must be sold for Company B's profits to be larger than Company A's profits?

2.5 Linear Inequalities and Systems of Linear Inequalities

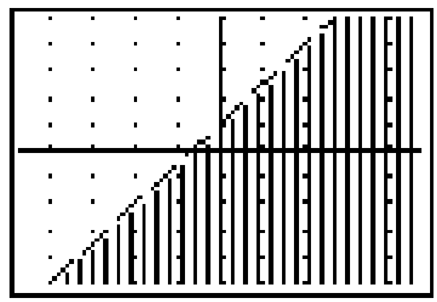
The solutions to linear inequalities must be shown graphically.

To graph:

1. Replace the inequality sign with an equal sign and graph the line.
2. Draw the line in solid if the inequality is ≤ or ≥. Draw a dotted line if the inequality is < or >.
3. Pick any point that is not on the line as a test point. Put the coordinates of this point into the inequality. If the inequality is true for this point, shade that side of the line. If the inequality is false, shade the opposite side.

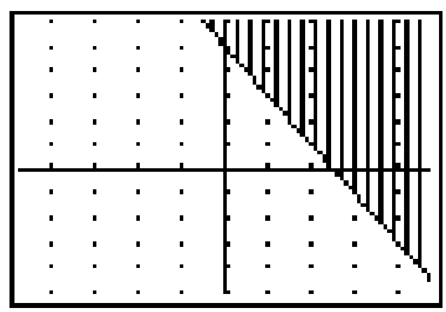
Examples:

1. Graph $y < 3/2\ x + 1$.



First graph the line $y = 3/2\ x + 1$.
The line will be drawn dotted since the inequality sign is <.
Picking a test point of (0, 2), gives $2 < 3/2(0) + 1$ which is false. We shade the side of the line that does not include the point (0, 2).
Note that if we picked (0, 0) the inequality yields $0 < 1$ which is true and we would shade the side with (0, 0). This has the same result.

2. Graph $2x + y \geq 5$.

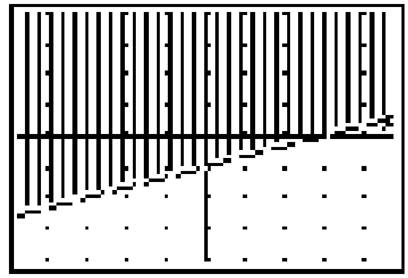


First graph the line $2x + y = 5$.

The line will be solid since the inequality is ≥. Picking a test point of (1, 0) gives $2 \geq 5$ which is false. So shade the side that does not include the point (1, 0).

To write the equation of an inequality given the graph, we first find the equation of the line. Choose a point in the shaded section of the graph and use the correct inequality sign for those coordinates. Look to see if the line is solid or dotted to decide whether the inequality includes an equal sign.

Example: Given the graph shown, write the inequality for the graph.



The graph has a slope of 1/3 and a y-intercept of (0, -1) so the equation of the graph is y = 1/3 x – 1. Choosing a point in the shaded section, we can use (0, 0). We know we need an inequality sign, to decide which is appropriate, replace the coordinates in the equation giving:

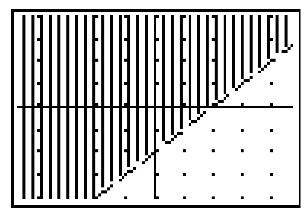
0 ☐ 1/3 (0) – 1

To make this a true statement, we must use >. The line is dotted so there is not an equal sign. The inequality is y > 1/3x – 1.

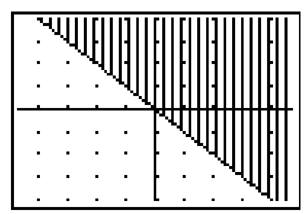
Systems of linear inequalities include multiple inequalities which must all be solved simultaneously. We are looking for the points which satisfy all the inequalities in the system. Graphically, this is the overlapping sections of the individual inequalities. To find the solution to a system, graph each individual inequality and identify the points that are shaded for all given inequalities. These points represent the solution to the system.

Example: Graph the solution to the system $\begin{cases} y > x - 2 \\ y \geq -x \end{cases}$.

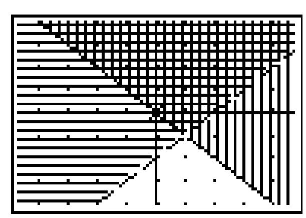
1. Graph y > x – 2. The slope is 1, the y-intercept is (0, -2), and the line will be dotted. Using (0, 0) as a test point, it satisfies the inequality so that side of the line will be shaded.



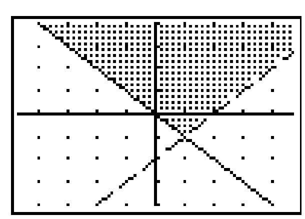
2. Graph y ≥ -x. The slope is -1, the y-intercept is (0, 0), and the line will be solid. Using (1, 0) as a test point, it satisfies the inequality so that side of the line will be shaded.



3. Both equations on the same graph looks like:



4. The solution includes all points which were shaded for both inequalities.



Homework:

Sketch graphs of the following inequalities.

1. y < -2x + 3

2. y ≤ -3x + 5

3. y ≥ ½ x + 1

4. y > 3/2 x – 4

5. 2x + 3y < 6

6. –x + 4y ≥ 8

Graph the solution to the system of inequalities.

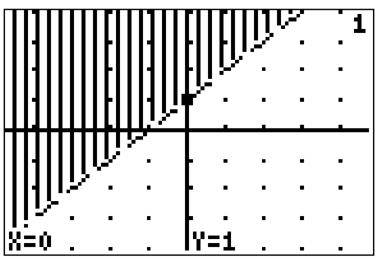
7. $\begin{cases} y > \frac{1}{3}x - 1 \\ y \leq 2 \end{cases}$

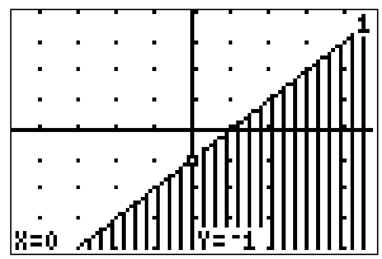
8. $\begin{cases} y \leq -\frac{4}{3}x + 2 \\ y \leq 2x \end{cases}$

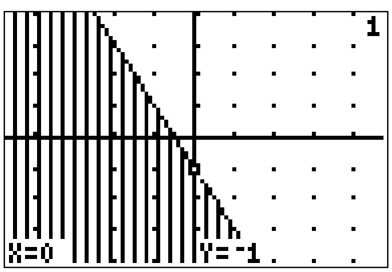
9. $\begin{cases} y > 3x - 1 \\ y \leq -\frac{1}{2}x + 2 \end{cases}$

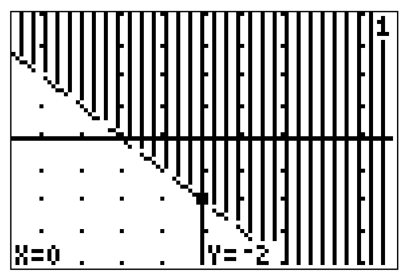
10. $\begin{cases} y < -\frac{2}{5}x + 3 \\ x \geq 0, y \geq 0 \end{cases}$

Write the inequality for the graph shown.

11.


12.


13.


14.


15. Sheila needs to stock the snack machine at work. Each bag of chips sells for \$0.75 and each candy bar sells for \$0.60. Sheila needs to make at least \$25 a month from the snack machine. Write a system of three inequalities for the number of chips and candy bars that must be sold and graph the solution.

16. Bill has 150 acres of farmland for growing soybeans or corn. He can make a profit of \$297 an acre for soybeans and \$412 an acre for corn. He wants to have a profit of at least \$55,000. Write a system of inequalities for the number of acres he can use for each crop and graph the solutions.

2.6 Systems of Linear Equations

A system of equations consists of two or more equations that are solved simultaneously. In this section, we will focus on systems of 2 linear equations. Systems can be solved graphically, numerically and algebraically.

Solving numerically:

To solve a system numerically, we are looking for the same coordinate point on both graphs. Make a table for both equations and find the point that the tables have in common.

Example:

1. Solve the system numerically. $\begin{cases} y = -x + 4 \\ y = 2x - 5 \end{cases}$

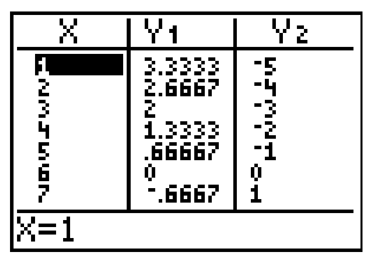
   Make a table for select x-values for both equations. We are looking for the point that both equations have in common.

x	-x+4	2x-5
-1	5	-7
0	4	-5
1	3	-3
2	2	-1
3	1	1

   Looking at the table, at x = 3, both equations have an output of 1. The solution to the system will be the point (3, 1).

2. Solve the system numerically using the graphing calculator to make the table. $\begin{cases} y = -\frac{2}{3}x + 4 \\ y = x - 6 \end{cases}$

   Letting y1 = $-\frac{2}{3}x + 4$ and y2 = x – 6. The table is shown. The point that the two equations share is (6, 0) so that is the solution.



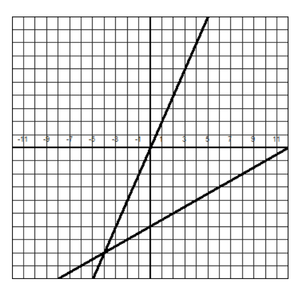
Solving graphically:

To solve a system graphically, we need to find the point that the graphs have in common or the intersection point. This may be done by hand or using technology and the intersection command.

Examples:

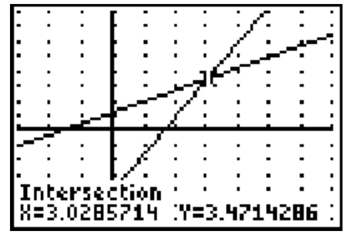
1. Solve the system graphically. $\begin{cases} y = \frac{1}{2}x - 6 \\ y = 2x \end{cases}$

   To solve graphically, first graph each line. Then, find the intersection point of the lines. Looking at the graph, we can see that the lines intersect at the point (-4, -8).



2. Solve the system graphically using the graphing calculator. $\begin{cases} y = 0.75x + 1.2 \\ y = 2.5x - 4.1 \end{cases}$

Set y1 = 0.75x + 1.2 and y2=2.5x – 4.1. Choose an appropriate viewing window so that you can see the point of intersection. Use the intersection command located under the CALC menu.



The solution is (3.029, 3.471).

Solving algebraically:

There are two algebraic methods to solve systems of equations. They are the substitution method and the elimination method. The method used often depends on the types of equations in the system and the difficulty of solving by the methods. For linear systems, both methods can be used but sometimes one is simpler to use than the other. Remember we are still looking for the point that solves both equations.

Substitution method:

Steps:

1. Solve one equation for one of the variables if this is not already done. Choose either variable from either equation. Pick the simplest one.
2. Substitute into the other equation for that variable.
3. Solve for the variable.
4. Use one of the equations to solve for the other variable.
5. Check your answer.

Examples:

1. Solve the system $\begin{cases} 3x + 2y = 4 \\ y = x - 8 \end{cases}$ using the substitution method.

The first step is done since the second equation is already solved for y. Now, substitute into the first equation x – 8 for y.

3x + 2(x – 8) = 4
3x + 2x -16 = 4
5x – 16 = 4
5x = 20
x = 4
Using the second equation to solve for y: y = 4 – 8 = -4. So the solution is (4, -4). Check the solution in the other equation. 3(4) + 2(-4) = 4 checks.

2. Solve the system using the substitution method. $\begin{cases} 2x - y = 11 \\ x + 4y = -2 \end{cases}$

The second equation can be easily solved for x. x = -4y – 2

Substituting into the first equation:

2(-4y – 2) – y = 11
-8y – 4 – y = 11
-9y – 4 = 11
-9y = 15
y = -5/3
Using the second equation to find x: x + 4(-5/3) = -2 so x = 14/3. The solution is (14/3, -5/3).
Checking the solution in the first equation 2(14/3) – (-5/3) = 11.

Elimination Method: The idea of the elimination method is to add the two equations together to eliminate one of the variables. In order for a variable to be eliminated, the coefficients must be additive opposites so that the coefficients add to zero for that variable. This may be achieved by multiplying one or both equations by constants.

Examples:

1. Solve the system $\begin{cases} 2x + 3y = 7 \\ -x - 3y = 2 \end{cases}$ by the elimination method.

The coefficients of y will add to zero so we do not need to alter the equations before adding them.

$$\begin{array}{l} 2\mathrm{x} + 3\mathrm{y} = 7 \\ -\mathrm{x} - 3\mathrm{y} = 2 \\ \hline \mathrm{x} \qquad\quad = 9 \end{array}$$

We must find the other variable to have the intersection point. Using the first equation,
2(9) + 3y = 7
18 + 3y = 7
3y = -11
y = -11/3
Be sure to check the solution of (9, -11/3).

2. Solve the system $\begin{matrix} 4x - 2y = 5 \\ x - 3y = 0 \end{matrix}$ by the elimination method.

Neither of the pairs of coefficients add to zero. If the second equation is multiplied by -4 then the coefficients of x will be opposites. $-4(x - 3y = 0)$ gives $-4x + 12y = 0$

$4x - 2y = 5$

$\underline{-4x + 12y = 0}$

$10y = 5$ so $y = ½$

Plugging this into the first equation, $4x - 2(½) = 5$ gives $x = 3/2$. The solution to the system is (3/2, 1/2).

Types of systems:

Systems can be classified in **two** ways. The classifications can be used to determine the number of solutions of a system and the relationship of the graphs of the equations.

Consistent or Inconsistent?

A system is **consistent** if the system has at least one solution. All of the above examples were consistent systems since they had one solution.

A system is **inconsistent** if the system has no solutions. Linear systems that represent parallel lines have no solution; two parallel lines will never cross each other and thus have no intersection point. A set of parallel lines will have the same slope but different y intercepts. For instance, consider the system $-2x + y = -3$ and $4x - 2y = 10$. Using algebra, you can rewrite these equations as $y = 2x - 3$ and $y = 2x - 5$. The first line has a slope of 2 and a y-intercept of -3; the second has a slope of 2 and a y-intercept of -5. Since these lines are parallel, the system has no solution and is inconsistent.

Dependent or independent?

A system is **independent** if the system contains unique equations.

A system is **dependent** if the equations represent the same graph. A dependent system will have infinite solutions. Consider the linear system $-9.1x + 2.8y = 7$ and $63.7x - 19.6y = -49$. At first, these may appear distinct equations; however, after you simplify them, you obtain $y = 3.25x + 2.5$ for both equations. Since they represent the same line, this set of equations has infinitely many solutions. For example, the points (0, 2.5), (2, 9) and (10, 35) are just three solutions to the system, though you can find infinitely many more.

When solving either an inconsistent or dependent system algebraically, both variables will be eliminated in the solution process leaving an equation with constants only such as $0 = 0$ or $2 = 5$. If the equation is true, the system is a dependent system and has infinitely many solutions. If the equation is false, the system is inconsistent and has no solution.

Example:

Solve the system algebraically. $\begin{array}{c} y = -2x + 1 \\ 6x + 3y = 8 \end{array}$

Solving the system with the substitution method:

Substituting for y in the second equation gives: 6x + 3(-2x + 1) = 8 which simplifies to 3 = 8. This statement is false and therefore, the system has no solution. The system is a set of parallel lines and is inconsistent and independent.

Systems of three linear equations

Systems of three equations are usually solved with the elimination method. The first step is to use two different pairs of the equations and eliminate the same variable from each pair. At this point, you will have two equation with two variables just like the previous problems in this section. Solve this system and be sure to find all the variables.

Example:

Solve the system: $\begin{cases} 2a + 3b - c = 1 \\ 3a - b + 2c = 14 \\ a - 2b + 3c = 11 \end{cases}$

For convenience sake, we number the equations.
1. 2a + 3b – c = 1
2. 3a – b + 2c = 14
3. a – 2b + 3c = 11

Now choose a variable to eliminate. If we choose to eliminate variable a, then choose two equations and eliminate that variable.
Using equation 1 and 3, we would multiply equation 3 by -2 to give:
2a + 3b – c = 1
<u>-2a + 4b – 6c = -22</u>
7b – 7c = -21 (equation 4)

Now choose a different pair and still eliminate variable a. If we use equation 2 and 3, we can multiply equation 3 by -3 to give:
3a – b + 2c = 14
<u>-3a + 6b – 9c = -33</u>
5b – 7c = -19 (equation 5)

Now using equation 4 and 5, we can eliminate variable c by multiplying equation 5 by -1.

7b – 7c = -21

<u>-5b + 7c = 19</u>

2b = -2 so b = -1

Then solving for c, we find c = 2. Using any of equations 1, 2, or 3, we can find a = 3.

Homework:

Solve the systems numerically.

1. $\begin{cases} y = 2x - 8.5 \\ y = -0.5x + 1.5 \end{cases}$

2. $\begin{cases} 2x + 3y = -1 \\ 3x + 2y = -4 \end{cases}$

Solve the systems graphically.

3. $\begin{cases} y = -x \\ y = 2x - 3 \end{cases}$

4. $\begin{cases} x - 2y = -2 \\ y = -x - 2 \end{cases}$

Solve the systems using the substitution method.

5. $\begin{cases} x + y = -3 \\ 2x - 3y = 19 \end{cases}$

6. $\begin{cases} y = x + 2 \\ 3x - 2y = -5 \end{cases}$

Solve the systems using the elimination method.

7. $\begin{cases} x + y = -3 \\ 2x - 5y = 20 \end{cases}$

8. $\begin{cases} 3x + 5y = 1 \\ 2x - 3y = 5 \end{cases}$

9. From 1990 through 1995, the number of radios sold (in millions) can be modeled by y = -0.3(x - 1990) + 21.6 and the number of televisions sold (in millions) can be modeled by y = 0.9(x – 1990) + 20.4. In which year were the same number of radios and televisions sold? Show or explain how you got your answer.

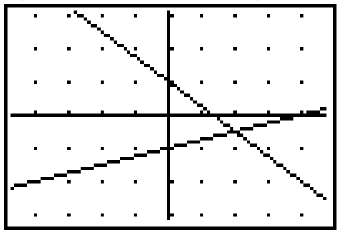
10. An office supply store sells 7 notebooks and 4 pens for $6.40. You could also purchase 2 notebooks and 19 pens for $5.40. Set up a system and solve to find the cost of each item.

11. I went to the store and bought three packages of bacon and two cartons of eggs and paid a total of $7.45. My husband, not knowing that I had already gone to the store, went to the same store and bought 2 packages of bacon and 3 cartons of eggs and paid $6.45.

   A. Set up a system of equations.

   B. Solve the system **algebraically** to find out how much money we would get back if we returned two packages of bacon and two cartons of eggs. Be sure to show the algebraic work.

12. Shown is the graph of a system of equations. Use the graph to estimate the solution to the system. Show on the graph or explain in words how you used the graph to find the solution. The scale on the axes is 1 unit.



13. Salespeople often are paid a small salary and then commission on their sales. If person A makes \$250 plus 10% of sales and person B makes \$550 plus 6% of sales, then find the level of sales such that the income earned is the same for both people.

14. One angle measures 15° more than the other. Find their measures if the angles are complementary.

15. The length and width of a photo add to 20 inches. The length is 4 inches longer than the width. Find the dimensions of the picture.

16. How much hydrogen peroxide in a 3% solution must be blended with water to obtain 1.5 liters of a 0.4% solution? How much water is needed?

17. How many pounds of dog food A, containing 16% protein, must be blended with dog food B, containing 5% protein, to obtain 500 pounds of a blend with 7% protein?

18. An inheritance of \$75000 is to be split and invested at 6% and 9% interest to produce an average of 8% interest. How much money should be invested at each rate?

19. Fitz earns \$1782 interest on a total of \$23600 placed in two investments. The investments earn 4.5% and 8.5%. How much money is in each investment?

Solve the systems by any method. Classify the system.

20. $\begin{cases} 2x-3=2y \\ x-y=6 \end{cases}$

21. $\begin{cases} x-2y=5 \\ 2x-10=4y \end{cases}$

22. $\begin{cases} y-2x=3 \\ y+x=-2 \end{cases}$

23. $\begin{cases} 4x+y=10 \\ 3x-2y=1 \end{cases}$

Solve the following systems.

24. $\begin{cases} 3x-2y-z=0 \\ 4x-3y+z=-17 \\ 5y+2z=-4 \end{cases}$

25. $\begin{cases} 3a-b+2c=39 \\ 2a-2b-3c=18 \\ 4a+3b+c=15 \end{cases}$